**Physics 1C, Summer Session I, 2011**
**Final Exam**

Instructions: Do any 8 problems. Please do all work on separate sheets of paper, and hand in everything you want to be graded. Clearly mark the problems that you want graded, and box the answers.

**1.** A mass on a spring undergoes simple harmonic motion. The speed of the mass is given by the following expression:

$$v(t) = -(0.35m/s)\sin\left[(12.6s^{-1})t\right]$$

(a) What is the frequency of the mass's motion?

(b) What is the total energy of the harmonic oscillator?

**Solution:**
(a) The angular frequency is $12.6s^{-1}$, so the frequency is $12.6s^{-1} / 2\pi = 2.01s^{-1}$.

(b) The total energy is equal to the kinetic energy when the speed is at its maximum value, since the potential energy there is zero. The maximum speed is 0.35m/s. Thus, $E = 1/2\ Mv^2_{MAX} = (0.0613m^2/s^2)M$.

Due to a typo, the mass *M* is missing from the problem statement; it's supposed to be 0.10*kg*. It's OK to just leave it as *M*, and if this mistake was discovered during the actual final, we would just announce a correction and write the value of *M* on the board.

**2.** A string with a mass per unit length of 0.050*kg/m* is tied down at both ends, 1 meter apart, and held under tension. The fundamental frequency of the string is 120*hz*.

(a) What is the 5th harmonic frequency?

(b) What would be the fundamental frequency if the tension was doubled?

**Solution:**
(a) The 5th harmonic frequency is just 5 times the fundamental, so it is 600*hz*.

(b) The fundamental frequency is proportional to the square root of the tension, so increasing the tension by a factor of 2 would increase it by a factor of sqrt(2). The new fundamental frequency would be 120 sqrt(2) = 170 *hz*.

**3.** A jet aircraft flying 300 meters overhead produces a sound intensity level of 110 decibels on the ground. What is the total power produced in sound waves, in watts?

(a) What is the intensity level of this sound, in decibels, when heard from a distance of 20 kilometers?

(b) What is the total power emitted by the aircraft in the form of sound waves?

**Solution:**
(a) 20*km* away is 20000 / 300 = 66.7 times farther than 300*m*. The intensity is thus lower by a factor of $66.7^2 = 4440$. The intensity level is lower by 10 $\log_{10}(4440)$ = 36db, so the new intensity level is 110db - 36db = 74db. The intensity level of the sound at a distance of 20 kilometers is 74 decibels.

(b) First solve for the intensity at 300*m*:

$$\beta = 110db = 10\log_{10}\left(\frac{I}{10^{-12}W/m^2}\right)$$

$$I = 10^{\beta/10} \times 10^{-12}W/m^2 = 0.1W/m^2$$

This intensity is spread over a sphere of radius 300 meters. The total power is therefore

$$P = IA = 4\pi R^2 I = 4\pi\left(300m^2\right)\left(0.1W/m^2\right) = 1.13 \times 10^5 W = 113kW$$

Thus the jet produces 113kW of power in sound waves.

**4.** When sound strikes a cliff, it is observed to be fully reflected unless the angle between the direction of the sound and the normal to the cliff surface is less than 6.2 degrees. For angles less than 6.2 degrees, some of the sound is absorbed by the cliff. If the speed of sound in air is 330 *m/s*, what is the speed of sound in the rock of the cliff?

**Solution:** The speed of sound is higher in the cliff than in air, so for angles greater than the critical angle of 6.2 degrees, the sound is fully reflected. From the equation for the critical angle,

$$\sin\theta_C = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{v_{Air}}{v_{Rock}}$$

$$v_{Rock} = \frac{v_{Air}}{\sin\theta_C} = \frac{330m/s}{\sin(6.2°)} = 3060m/s$$

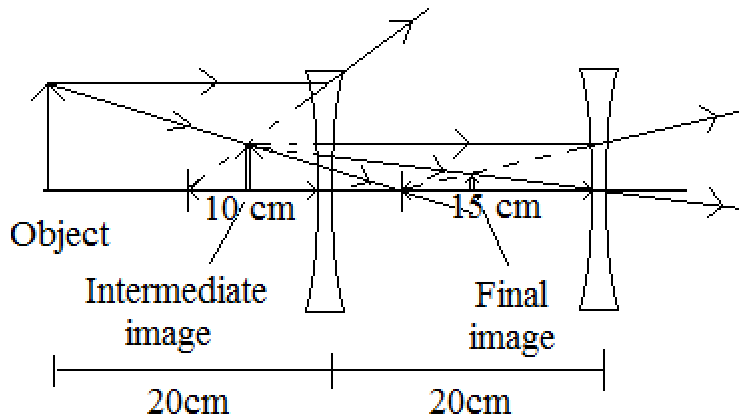
The speed of sound in the rock is thus 3060 m/s.

**5.** An object is placed 20*cm* in front of a divergent lens with a focal length of -10*cm*. A second divergent lens, with a focal length of -15*cm*, is placed 20*cm* behind the first lens.

(a) Draw a ray diagram for this situation.

(b) Calculate the position of the final image and the magnification.

**Solution:**
(a)




(b) First find the position and magnification of the intermediate image:

$$q_1 = \frac{1}{1/f_1 - 1/p_1} = \frac{1}{1/(-10cm) - 1/20cm} = -6.67cm$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{-6.67cm}{20cm} = 0.333$$

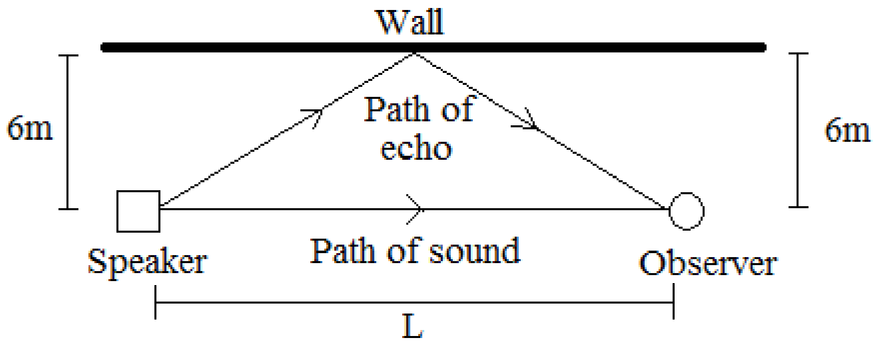
The intermediate image is 6.67cm in front of the first lens, so it is 26.67cm in front of the second lens. This is the object distance for the second lens. The image distance and magnification for the second lens is

$$q_2 = \frac{1}{1/f_2 - 1/p_2} = \frac{1}{1/(-15cm) - 1/26.67cm} = -9.60cm$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{-9.60cm}{26.67cm} = 0.360$$

The final image is located 9.60*cm* in front of the second lens. The overall magnification is $M = M_1M_2$ = 0.333 x 0.360 = 0.120.

**6.** A loudspeaker is located some distance *L* from an observer and 6 meters away from a sound-reflecting wall. The observer is also 6 meters away from the wall. When the sound reflects from the wall, it undergoes no phase change since the speed of sound in the wall is higher than in air. If the sound from the loudspeaker has a frequency of 120*hz*, at what distances from the loudspeaker is the sound quieter due to destructive interference between the sound and the echo? Give all locations for $L < 20$ meters. The speed of sound in air is 330 *m/s*.




**Solution:**

The path length of the sound coming directly from the speaker is *L*, while the path length of the echo is

$$2\sqrt{(L/2)^2 + (6m)^2} = \sqrt{L^2 + 144m^2}$$

For destructive interference, the path length difference must be a half-integer multiple of the wavelength:

$$\sqrt{L^2 + 144m^2} - L = \left(j + \frac{1}{2}\right)\lambda$$

$$L^2 + 144m^2 = \left(\left(j + 1/2\right)\lambda + L\right)^2 = \left(j + 1/2\right)^2 \lambda^2 + L^2 + 2\left(j + 1/2\right)\lambda L$$

$$144m^2 = \left(j + 1/2\right)^2 \lambda^2 + 2(j + 1/2)\lambda L$$

$$L = \frac{144m^2 - (j + 1/2)^2\lambda^2}{2(j + 1/2)\lambda}$$

Calculate $\lambda$: $\lambda = c / f = 330\text{m s}^{-1} / 120\text{ s}^{-1} = 2.75m$. Therefore

$$L = \frac{144m^2 - (j + 1/2)^2\lambda^2}{2(j + 1/2)\lambda} = \frac{52.4 - 2.75(j + 1/2)^2}{2(j + 1/2)}m$$

Running through different values of *j*, starting from zero, we obtain distances of: 51.7*m*, 15.40*m*, 7.04*m*, 2.67*m*. The last three are under 20*m* away. Increasing the value of *j* further gives spurious negative values of *L*.

**7.** Unstable particles are produced traveling with a speed of 0.962*c*. The particles travel an average distance of 1.2*cm* from the source before decaying. What is the lifetime of the particles in the frame where they are at rest?

**Solution:** If the time of the particle in its rest frame is $t_0$, then the lifetime in the lab frame is $\gamma t_0$, and the average distance traveled in the lab is $d = v\gamma t_0$. Therefore,

$$t_0 = \frac{d}{\gamma v} = \frac{1.2cm\sqrt{1-0.962^2}}{0.962 \times 3 \times 10^{10} cm/s} = 1.14 \times 10^{-11} s$$

**8.** Two beams of protons collide head-on in a particle accelerator. In the lab frame, the protons in each beam have a kinetic energy of 4.00 GeV. The rest energy of a proton is 0.938 GeV.

(a) Find the speed of the protons relative to the lab, as a fraction of *c*.

(b) Find the speed of the protons in one beam relative to the protons in the other beam.

**Solution:**
(a) The speed can be found as follows:

$$E = E_K + mc^2 = 4.00GeV + 0.938GeV = 4.938GeV$$

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}} \qquad 1 - \frac{v^2}{c^2} = \frac{m^2c^4}{E^2}$$

$$\frac{v}{c} = \sqrt{1 - \frac{m^2c^4}{E^2}} = \sqrt{1 - \left(\frac{0.938GeV}{4.938GeV}\right)^2} = 0.9818$$

The protons move at 0.9818*c*.

(b) The photons both have a speed *v* relative to the lab, as found above. They are moving head-on towards each other, so their speed relative to each other is

$$v' = \frac{v+v}{1+vv/c^2} = \frac{2 \times 0.9818c}{1+0.9818^2} = 0.9998c$$

The protons move at 0.9998*c* relative to each other.

**9.** A beam of electrons is accelerated through a voltage *V*. The beam is incident on a crystal with lattice spacing of 0.22*nm*. The angle between the central maximum and the first diffraction peak on the screen is 16 degrees. What is the accelerating voltage *V*?

First we find the de Broglie wavelength of the electron from the equation for the diffraction angle of the first diffraction peak:

$$d\sin\theta = \lambda \qquad \lambda = d\sin\theta = (0.22nm)\sin(16°) = 0.0606nm$$

Now we find the kinetic energy:

$$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2c^2}{2(mc^2)\lambda^2} =$$

$$= \frac{(4.14\times 10^{-15}eV\,s)^2(3\times 10^8 m/s)^2}{2(5.11\times 10^5 eV)(6.06\times 10^{-11}m)^2} = 411eV$$

To get 411eV of kinetic energy, the electron must be accelerated through a potential of 411 volts.

**10.** A muon decays via the weak interaction into three particles. One of these is an electron.

(a) What must the other two particles be? Use conservation of lepton number for each generation of leptons.

(b) Draw a Feynman diagram for this process.

**Solution:**

(a) The muon is not in the final state, so it must be replaced by a muon neutrino to conserve the muon lepton number. The electron lepton number is zero in the initial state, but an electron is present in the final state, so an electron antineutrino must also be present so that the net electron lepton number remains zero.

Thus the other two particles are a muon neutrino and an electron antineutrino.

(b)